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Nonlinear Phenomena in Multi-stable Structures Dynamics

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Abstract:
Multi-stability is a characteristic feature of nonlinear systems, and it is observed by the existence of a few solutions for the same set of structural parameters. A classical Duffing's oscillator is an example of the model where three different periodic solutions, two stable and one unstable, may exist in a certain frequency domain. However, if the Duffing oscillator is excited parametrically and at the same time exited externally it is possible to get five solutions, two stable and three unstable as presented in [1]. Moreover, self-excitation creates an additional quasi-periodic solution which occurs after Neimar-Sacker bifurcation [1]. The models governed by classical Duffing's equation have just one equilibrium, which is stable and the corresponding potential function possesses just one minimum. But in some systems more than one equilibrium may exist. Multi-stability can be created by dedicated devices, for example by creating repulsing magnetic force or by a specific design of elastic elements of a nonlinear oscillator. Such nonlinear force enables modifying the shape of a potential function and to get two or more potential wells. Another option is to add axial force to the structure which transfers the vibrating system close to a buckling point with two equilibria (two potential wells). This effect can be caused by axial mechanical loading or by increased temperature, for example.

Composite technology offers new possibilities to create multi-stable structures. A special design of laminated shells based on a unique conical or pseudo-conical shape, with asymmetric configuration of lamina is proposed in [2]. Depending on the assumed geometry two or even five stable equilibria can be obtained. The unique nonlinear properties of the proposed shells are investigated for local (in-well) oscillations and as well as for large oscillations with global (cross-well) dynamics which include the snap-through effect. The main goal of this paper is to present the untypical nonlinear effects of multi-stable systems and their application to energy harvesting [3], morphing and control.

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