A Superposition-Based Framework for Identifying Multiple Cracks in Elastic Structures

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Abstract:
A superposition-based framework is presented for identifying multiple breathing cracks in elastic structural components, offering a computationally efficient inverse-analysis technique. The approach builds on the observation that localized damage reduces the beam's capacity to store energy, resulting in proportional reductions in modal frequencies. A crack produces, for any bending vibration mode, a distinct decrease in frequency that depends on the beam curvature at the crack location for that mode. If we consider the beam a conservative system, the modal stored energy is the same in the opening and closing phases. Because the frequency decrease due to the crack is proportional to the loss of stored energy, in the case of a single crack, the first several frequency shifts provide a clear damage signature, uniquely associated with the crack location and severity. It is convenient to normalize frequency changes to make them more comparable. In prior research, we successfully formulated the relationship between the frequency of the damaged beam and the severity and location of the crack using an algebraic equation.
In this presentation, we demonstrate the effectiveness of superposition in assessing multiple cracks in a beam. In most research, the vibration of a beam with a breathing crack is treated as nonlinear, so applying the superposition of crack effects to determine the locations and severity of multiple cracks is considered unreliable. Indeed, the displacement of a beam with a breathing crack is bi-linear. Therefore, we use the energy losses to characterize the cracks, as they can be superposed. The assessment method consists of precomputing a library of elemental crack-induced perturbations—each corresponding to a crack of given depth and location—so that the global response of a structure containing an arbitrary combination of cracks can be rapidly synthesized. An optimization procedure or AI-supported technique is employed to determine the number, positions, and severities of cracks that best reproduce the measured response. Numerical experiments demonstrate that the approach remains robust to measurement noise and modeling uncertainties while requiring significantly fewer computational resources than full-scale iterative finite-element updating. Some particular cases, such as non-ideal fixing conditions or closely located cracks, are also discussed. The results indicate that the superposition-based strategy provides a reliable and scalable tool for structural health monitoring, enabling rapid detection of multiple damage sites and supporting real-time assessment of engineering structures.