

COUPLED KURAMOTO OSCILLATORS: GLOBAL DYNAMICS

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ABSTRACT.

A classical model for coupled vibrations are all-to-all coupled, identical Kuramoto oscillators

$$\dot{\vartheta}_j = \frac{1}{N} \sum_{k=1}^N \sin(\vartheta_k - \vartheta_j).$$

Stable synchrony has been known for half a century. Transients, however, involve many metastable 2-cluster states, and shift-type heteroclinic orbits among them. We present a mathematical description of the resulting global dynamics. Strong structural stability properties imply robustness under general (small) perturbations, including second order systems and delays. We promise a gentle introduction, for non-mathematicians and friends. This is joint work with Jia-Yuan Dai and Alejandro López-Nieto; see <https://arxiv.org/abs/2512.02937>.

HELP please: the main gap of our current understanding, curiously, involves the invariant set of closed *mechanical linkage chains*

$$0 = \sum_k \exp(i\vartheta_k).$$